



CRANBROOK  
SCHOOL

Centre Number

1 2 5

Student Number

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2016

HSC Trial Examination  
Assessment Task 3

# Extension 1 Mathematics

Reading time	5 minutes
Writing time	2 hours
Total Marks	70
Task weighting	40%

## General Instructions

- Write using a black pen
- A Board-approved calculator may be used
- A BOSTES formula sheet is provided
- Use the Multiple-Choice Answer Sheet provided
- All relevant working should be shown for each question 11-14

## Additional Materials Needed

- BOSTES Formula Sheet
- Multiple Choice Answer Sheet
- 4 writing booklets

## Structure & Suggested Time Spent

### Section I

#### Multiple Choice Questions

- Answer Q1 – 10 on the multiple choice answer sheet
- Allow about 17 minutes for this section

### Section II

#### Extended response Questions

- Attempt all questions in this section in a separate writing booklet
- Allow about 103 minutes for this section

This paper must not be removed from the examination room

### Disclaimer

*The content and format of this paper does not necessarily reflect the content and format of the HSC examination paper.*

# Section I

10 Marks

Allow about 17 minutes for this section

Use the multiple choice answer sheet for Questions 1-10.

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## Question 1

If  $N = 65$  when  $t = 0$ , then the solution to  $\frac{dN}{dt} = 0.3(N - 20)$  is:

- (A)  $N = 20 + 45e^{0.3t}$
- (B)  $N = 20 + 45e^{-0.3t}$
- (C)  $N = 20 - 45e^{0.3t}$
- (D)  $N = 20 - 45e^{-0.3t}$

## Question 2

In how many ways can the letters of the word SUCCESS be arranged?

- (A)  ${}^7C_7$
- (B)  $7!$
- (C) 420
- (D)  ${}^7P_7$

**Question 3**

A general solution to  $2\sin^2 x - 1 = 0$

(A)  $x = 2\pi n \pm \frac{\pi}{4}$

(B)  $x = \pi n \pm \frac{\pi}{4}$

(C)  $x = \pi n + (-1)^n \frac{\pi}{4}$

(D)  $x = \frac{\pi}{4}, \frac{3\pi}{4}$

**Question 4**

When  $P(x) = 5 - 3x^2 - x^3$  is divided by  $(x + 2)$ , the remainder is:

(A) 5

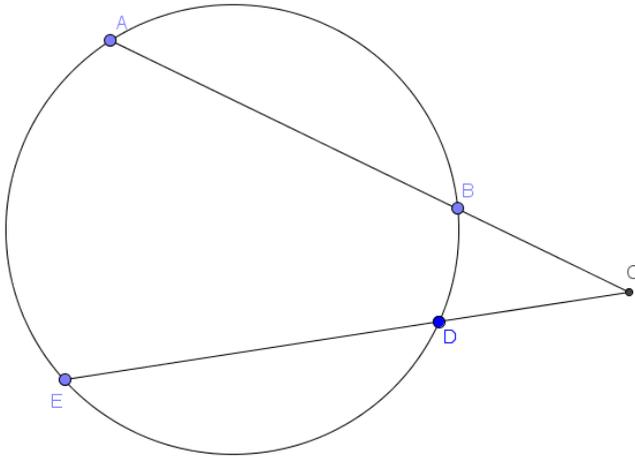
(B) -15

(C) 25

(D) 1

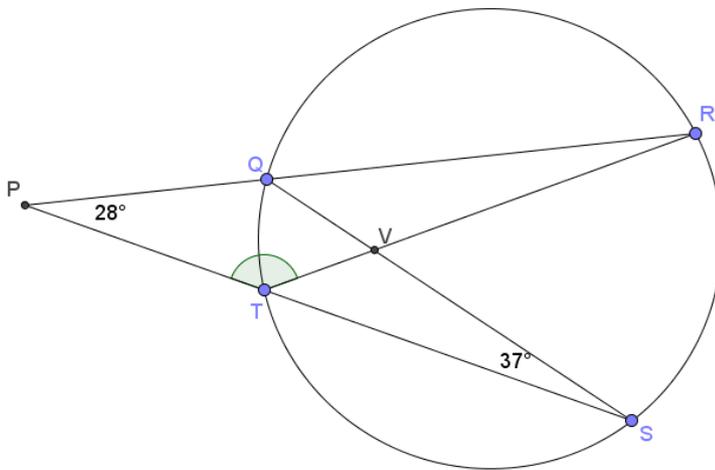
### Question 5

Select the statement which is true for the given diagram:



- (A)  $AB \times BC = ED \times DC$
- (B)  $AC \times BC = EC^2$
- (C)  $AC \times BC = EC \times DC$
- (D)  $AB \times ED = BC \times DC$

**Question 6**



**Diagram not to scale**

In the diagram above,  $\angle RPT = 28^\circ$  and  $\angle TSV = 37^\circ$ .

Find the value of  $\angle PTR$

- (A)  $143^\circ$
- (B)  $115^\circ$
- (C)  $74^\circ$
- (D)  $37^\circ$

**Question 7**

In terms of  $t$ , where  $\tan \theta = \frac{2t}{1-t^2}$ ,  $\frac{\cot \frac{\theta}{2} + \tan \frac{\theta}{2}}{\cot \frac{\theta}{2} - \tan \frac{\theta}{2}}$  can be expressed as:

- (A)  $\frac{1+t^2}{1-t}$
- (B)  $\frac{1+t^2}{1-t^2}$
- (C) 1
- (D)  $\frac{1-t^2}{1+t^2}$

**Question 8**

A particle is moving in Simple Harmonic Motion. Given  $v^2 = 6 + 4x - 2x^2$ , where  $v$  is the velocity of the particle and  $x$  is the displacement of the particle from the origin, the centre of motion is:

- (A)  $x = -1$
- (B)  $x = 2$
- (C)  $x = 1$
- (D)  $x = \sqrt{2}$

**Question 9**

What is the range of  $f(x) = 5 \cos^{-1} x$

- (A)  $0 \leq f(x) \leq \pi$
- (B)  $-5 \leq f(x) \leq 5$
- (C)  $0 \leq f(x) \leq \frac{1}{5}$
- (D)  $0 \leq f(x) \leq 5\pi$

**Question 10**

What is the value of  $\lim_{x \rightarrow 0} \frac{5 \sin 3x}{3 \sin 4x}$

(A)  $\frac{5}{3}$

(B)  $\frac{3}{4}$

(C)  $\frac{5}{4}$

(D) 1

**END OF SECTION I**

## Section II

60 Marks

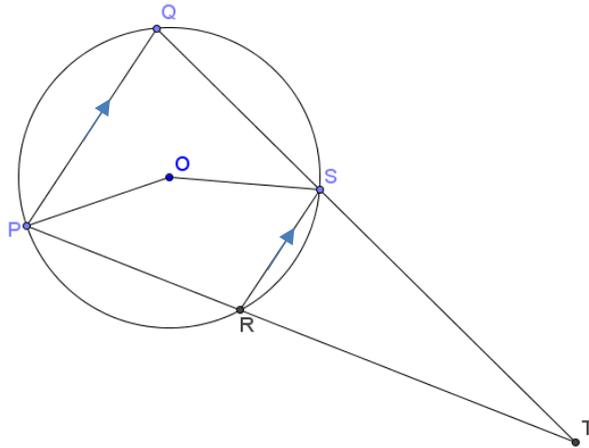
Allow about 103 minutes for this section

Answer questions 11-14 in separate booklets.

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Question 11	Begin a new booklet	15 Marks
a)	Find $\int 3\cos^2 3x \, dx$ .	2
b)	Solve $\frac{2x+1}{x-3} \leq 3$ .	3
c)	Two lines make an angle of $45^\circ$ with one another. If one line has a gradient of 2, what are the possible gradients of the other line?	2
d)	Use the substitution $u = 2x+6$ to find $\int x\sqrt{2x+6} \, dx$ .	2
e)	Given $A(2,3)$ and $P(11,18)$ , find the coordinates of $B$ , given $P$ divides $AB$ externally in the ratio $3:2$ .	2
f)	Given the polynomial $P(x) = x^5 - 7x^3 - 6x^2$ :	
	i. Explain why $P(x)$ is monic.	1
	ii. Find all zeros of $P(x)$ .	2
	iii. Sketch $P(x)$ .	1

End of Question 11



- a) In the diagram below, the points  $P, Q, R, S$ , lie on the circumference of the circle with centre  $O$ .  $PQ \parallel RS$

Prove:

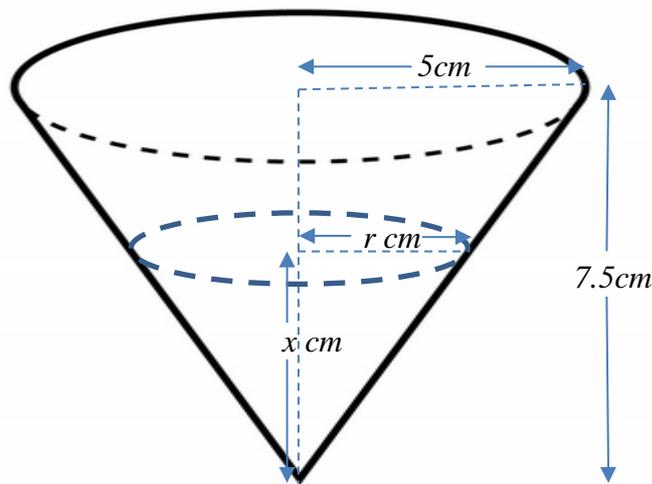
- |  |          |
|--|----------|
| i. $RT = TS$ .                         | <b>2</b> |
| ii. $PR = QS$ .                        | <b>1</b> |
| iii. $OPTS$ is a cyclic quadrilateral. | <b>2</b> |

- b) The equation of  $e^x = x + 2$  has a root close to  $x = 1.2$
- |   |          |
|---|----------|
| i. Using Newton's method, find a closer approximation to this, giving your answer correct to 2 decimal places.  | <b>2</b> |
| ii. If Newton's Method was used with an initial approximation $x = 0$ , the method would fail to provide a closer approximation of the root. Explain why it fails. Show all working | <b>1</b> |

**Question 12 continues on next page**

- c) Eight people attend a restaurant for dinner. They are provided with 2 circular tables, one of which seats 5 and the other 3.
- How many different seating arrangements are there? 2
  - If the seating is arranged at random, what is the probability that a couple find themselves on different tables? 2
- d) A filter paper is in the form of a cone, base radius 5 centimetres and perpendicular height of 7.5 centimetres.

The filter paper is inverted and filled with water. The water flows out at a constant rate of 1.5 centimetres cubed per second. At any given time, the depth of the water from the apex is  $x$  centimetres and the radius is  $r$ .



- Using similar triangles, show  $V = \frac{4}{27} \pi x^3$ .  
(Use the volume of a cone =  $\frac{1}{3} \text{Base Area} \times \text{Height}$ ) 1
- Find the rate at which the level of liquid is falling when the depth,  $x$ , is 5 cm. Give your answer correct to 2 decimal places. 2

**End of Question 12**

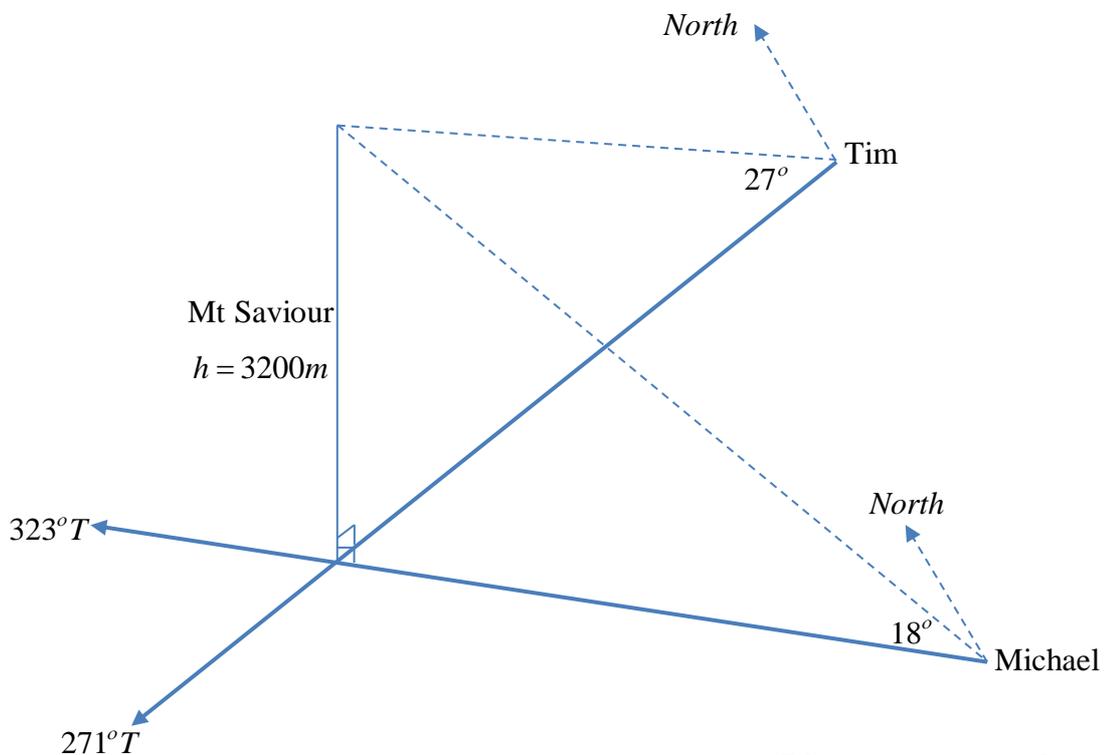
**Question 13****Begin a new booklet****15 Marks**

- a) Prove that  $\tan 2x + \cot 2x = 2 \operatorname{cosec} 4x$ . **2**
- b)
- i. Express  $2 \cos 2\theta + 3 \sin 2\theta$  in terms of one trigonometric ratio. **2**
- ii. Hence or otherwise solve:  $2 \cos 2\theta = 1 - 3 \sin 2\theta$ , for  $0 \leq \theta \leq 180^\circ$ .  
Answer to the nearest minute. **2**
- c) Prove by mathematical induction that  $9^{n+2} - 4^n$  is divisible by 5 for integers  $n \geq 1$ . **3**
- d) Prove  $\frac{d}{dx} \tan^{-1} x = \frac{d}{dx} \left[ -\tan^{-1} \left( \frac{1}{x} \right) \right]$ . **2**

**Question 13 continues on next page**

e) Tim is lost in the forest and Michael is searching for him. They are in contact via mobile phone. Tim and Michael can both see the top of Mt Saviour.

From Michael's position, the mountain has a bearing of  $323^\circ$ , and the angle of elevation to the top of the mountain is  $18^\circ$ . From Tim's position the mountain has a bearing of  $271^\circ$  and an angle of elevation to the top of the mountain of  $27^\circ$ . The top of Mt Saviour is  $3200m$  above sea level. Both Tim and Michael are at sea level.



**Diagram not to scale**

i. Show that the distance  $d$ , from Michael to Tim, can be found using:

$$d = \sqrt{3200^2 (\tan^2 63^\circ + \tan^2 72^\circ - 2 \tan 63^\circ \tan 72^\circ \cos 52^\circ)} \quad 2$$

ii. Find  $d$  to the nearest metre. 1

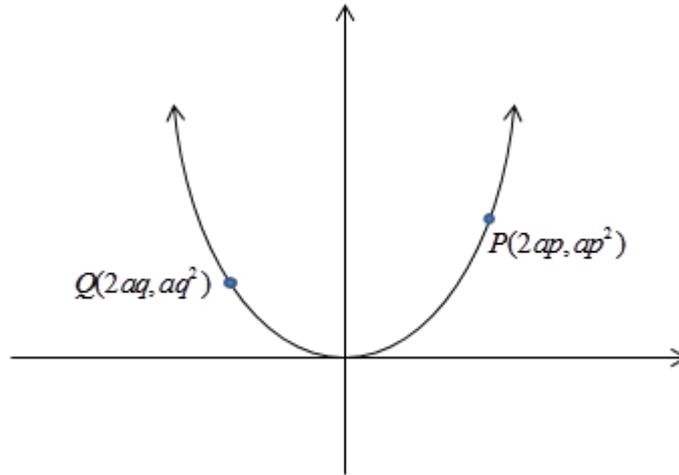
iii. At what bearing (to the nearest degree) must Michael walk to find Tim? 1

**End of Question 13**

**Question 14 Begin a new booklet**

**15 Marks**

- a)  $P(2ap, ap^2)$  and  $Q(2aq, aq^2)$  are two variable points on the parabola  $x^2 = 4ay$ .



- i. If the variable chord  $PQ$  is always parallel to the line  $y = x$  show that  $p + q = 2$ . **2**
  
- ii. The equation of the normal at  $P$  is  $x + py = 2ap + ap^3$  (You do not need to prove this). The normals at  $P$  and  $Q$  meet at  $N$ . Prove that the locus of  $N$  is a straight line. **3**
  
- b) A projectile is fired with an initial speed of  $56m/s$  and just clears a  $15m$  high wall which is  $70m$  from the point of projection. Let  $g = -9.8 m/s^2$ .
  - i. Show that:
 
$$x = 56 t \cos \theta$$

$$y = -4.9 t^2 + 56 t \sin \theta$$
 **2**
  - ii. At what possible angles could the projectile have been fired? **3**
  - iii. Explain why there are 2 answers. **1**

**Question 14 continues on next page**

c) The rise and fall of the tide at the mouth of a river is in simple harmonic motion.

The depth of water at low tide on a particular day is  $0.7m$  and the depth of water at high tide is  $3.7m$ .

Low tide occurs at  $8:55am$  and high tide is at  $3:05pm$ .

i. Show that  $n = \frac{\pi}{370}$ . **1**

ii. Find the earliest time at which a boat could enter if it requires the water to be at least 2 metres deep. **3**

**End of Question 14**

**END OF SECTION II**

**END OF EXAM**

$$1.) N = 20 + 45e^{0.3t}$$

$$\frac{dN}{dt} = 0.3 \times 45e^{0.3t}$$

$$= 0.3(N - 20)$$

(A)

2.) Success

$$\frac{7!}{2!3!} = 420$$

(C)

$$3.) \sin x = \frac{1}{\sqrt{2}}$$

$$x = \pi n \pm \frac{\pi}{4}$$

(B)

$$4.) P(x) = 5 - 3x^2 - 2x^3$$

$$P(-2) = 5 - 3(4) + 8$$

$$= -7 + 8$$

$$= \underline{\underline{1}}$$

(D)

5) (C)

$$6) 180 - 65 = 115^\circ$$

(B)

$$7) \frac{\frac{1}{t} + t}{\frac{1}{t} - t}$$

$$\frac{1+t^2}{1-t^2}$$

(B)

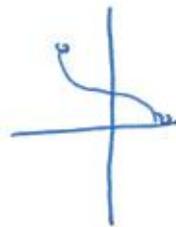
$$8) u^2 = -2(x^2 - 2x + 3)$$

$$= -2(x-3)(x+1)$$

centre = 12 

(C)

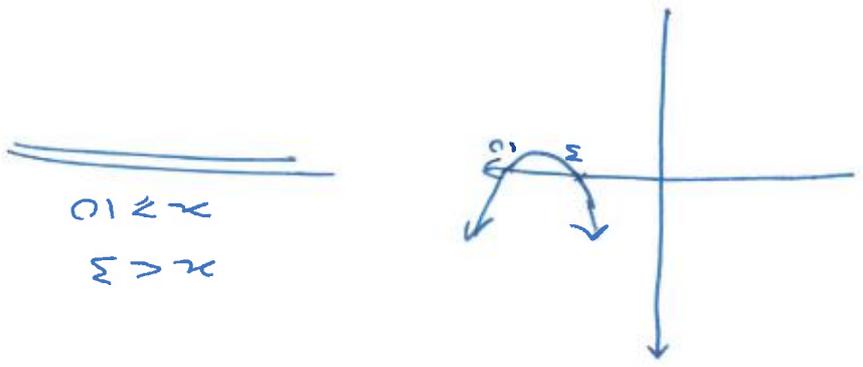
$$9) f(x) = 5 \cos^{-1} x$$



$$\underline{\underline{0 \leq f(x) \leq 5\pi}}$$

(D)





$$0 \leq (x-3)(x-10)$$

$$\leq \frac{3}{2}(x-3) [3(x-3) - (2x+1)]$$

$$0 \leq 3(x-3)^2 - (2x+1)(x-3)$$

$$(2x+1)(x-3) \leq 3(x-3)^2$$

$$\frac{2x+1}{x-3} \leq 3 \quad x \neq 3$$

$$= \frac{2}{3}x + \frac{1}{3} \sin 6x + c$$

$$= \frac{2}{3} \left( x + \frac{1}{6} \sin 6x \right) + c$$

$$\frac{2}{3} \int (1 + \cos 6x) dx$$

$$\int 3 \cdot \frac{1}{2} (1 + \cos 6x) dx$$

$$\cos^2 3x = \frac{1}{2} (1 + \cos 6x)$$

$$\cos^2 \theta = \frac{1}{2} (1 + \cos 2\theta)$$

$$\cos 2\theta = 2 \cos^2 \theta - 1$$

$$\text{ii) } \int 3 \cos^2 3x dx$$

$$c) \quad \tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$\tan 45^\circ = \left| \frac{m_1 - 2}{1 + 2m_1} \right|$$

$$1 = \left| \frac{m_1 - 2}{1 + 2m_1} \right|$$

$$\textcircled{1} \quad 1 + 2m = m - 2$$

$$\underline{\underline{m = -3}}$$

$$\textcircled{2} \quad -(1 + 2m) = m - 2$$

$$-1 - 2m = m - 2$$

$$1 = 3m$$

$$\underline{\underline{m = \frac{1}{3}}}$$

$$d) \quad \int x \sqrt{2x+6} \, dx$$

$$u = 2x + 6 \Rightarrow \frac{u-6}{2}$$

$$\frac{du}{dx} = 2$$

$$\int \frac{u-6}{2} \times \sqrt{u} \times \frac{du}{2}$$

$$\frac{1}{4} \int u^{3/2} - 6u^{1/2} \, du$$

$$\frac{1}{4} \left[ \frac{2u^{5/2}}{5} - \frac{2 \cdot 6 u^{3/2}}{3} \right] + C$$

$$\frac{2u^{5/2}}{20} - u^{3/2} + C$$

$$\underline{\underline{\frac{u^{5/2}}{10} - u^{3/2} + C = \frac{(2x+6)^{5/2}}{10} - \frac{(2x+6)^{3/2}}{10} + C}}$$

$$e) A(2, 3) \quad P(11, 18)$$

$$A(x, y) \quad P(x, y)$$

$$z: -2$$

$$\left( \frac{3x - 4}{1}, \frac{3y - 6}{1} \right)$$

$$3x - 4 = 11$$

$$3y - 6 = 18$$

$$3x = 15$$

$$3y = 24$$

$$\underline{\underline{x = 5}}$$

$$\underline{\underline{y = 8}}$$

$$f) P(x) = x^5 - 7x^3 - 6x^2$$

Coefficient of leading term is 1.

$$ii) P(x) = x^2(x^3 - 7x - 6)$$

$$P(-1) = \cancel{x^2}(-1 + 7 - 6)$$

$$= 0$$

$$P(x) = x^2(x+1)(x^2 - x - 6)$$

$$= x^2(x+1)(x^2 - x - 6)$$

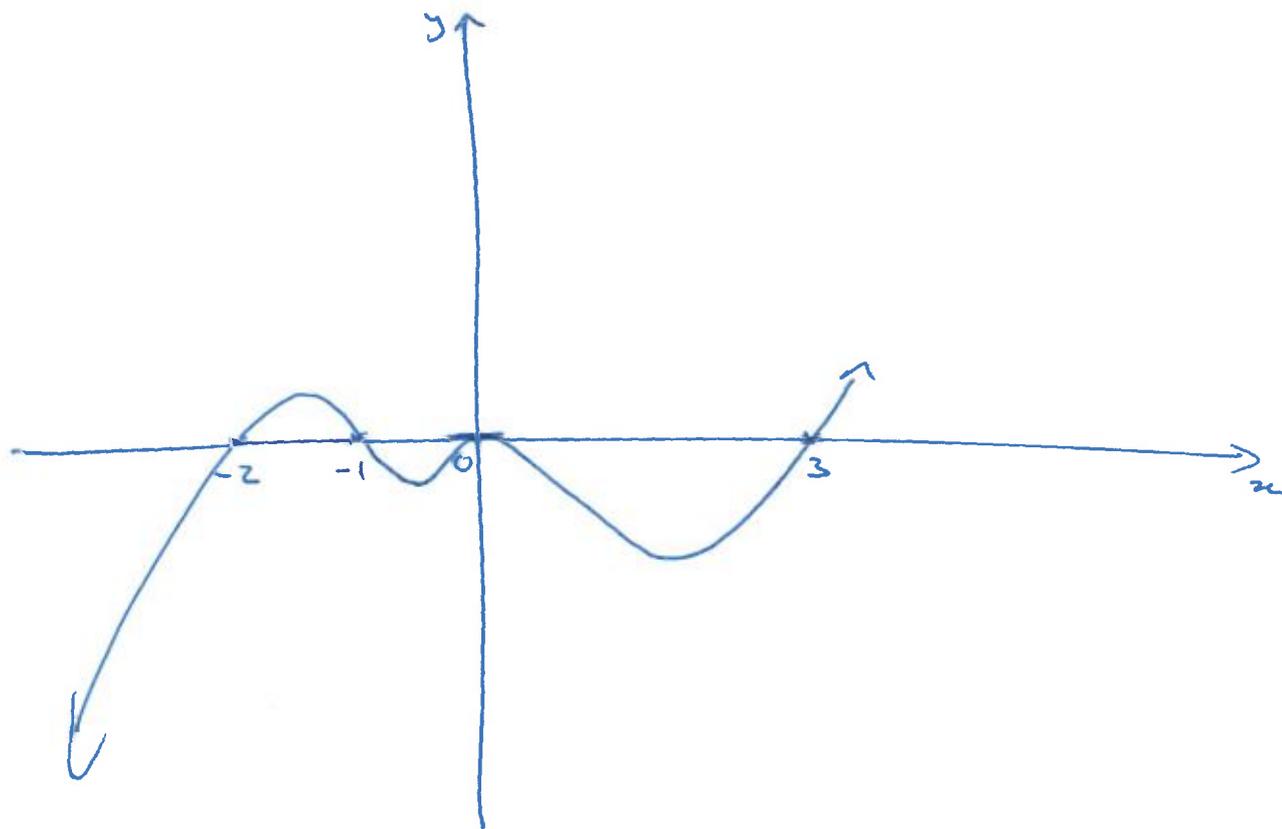
$$P(x) = x^2(x+1)(x-3)(x+2)$$

$$x = 0, -1, -2, 3$$

check

$$\cancel{(x-1)(x^2+x+6)}$$

$$\cancel{x^3 - x^2 + x^2 - x + 6x - 6}$$



$$12.) a) \text{ let } \angle PQS = \theta$$

$$\angle PRS = 180 - \theta \quad (\text{Opp. angles of cyclic quad})$$

$$\angle SRT = \theta \quad (\text{Angles on straight line})$$

$$\angle RST = \theta \quad (\text{Corresponding angles on parallel lines})$$

$$\therefore RT = ST \quad RST \text{ isosceles } \Delta.$$

$$PT \times RT = QS \times ST$$

$$RT = ST$$

$$\therefore PT = QT$$

$$(PR + RT) = QS + ST$$

$$\underline{PR = QS}$$

$$\angle POS = 2\theta \quad \text{Angle at centre, twice that at circumference.}$$

$$\angle STR = 180 - 2\theta \quad \text{Angles in a } \Delta$$

$$\therefore \text{As } \angle POS + \angle STR = 180^\circ \quad \text{POST is a cyclic quad} \\ (\text{Opp angles supplementary}).$$

$$6) \quad e^2 = x + 2 \quad \text{let } f(x) = e^x - x - 2 \quad f(1.2) = 0.135 \dots$$

$$f'(x) = e^x - 1$$
$$f'(1.2) = 2.32 \dots$$
$$f(x) = e^x - x - 2$$

$$f(x) = e^x - x - 2$$

$$= 1.2 - 0.058 \dots$$

$$\underline{= 1.142}$$

$$= \frac{15}{28}$$

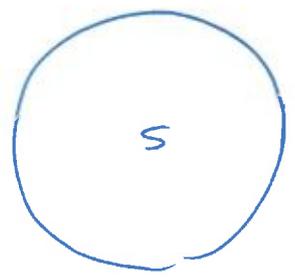
$$= \frac{1440}{2680}$$

$$= \frac{1440}{2680}$$

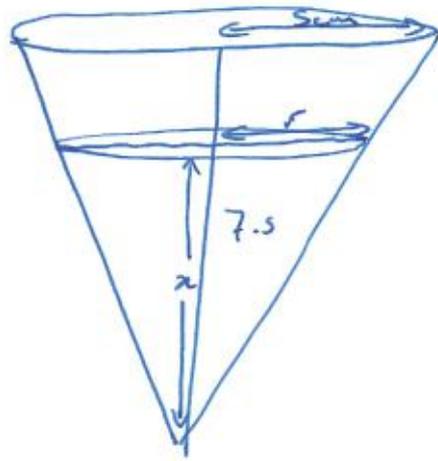
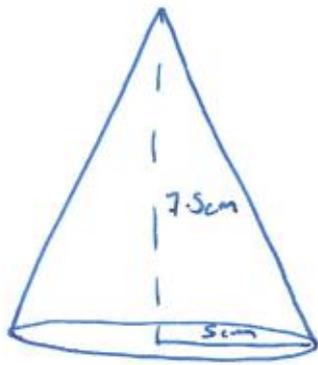
b) Place A on table of S, B on Z.  
 ∴  ${}^6C_4 \times 4! \times 2$   
 Choose other 4 for table of S.  
 switch tables for A and B.

for first table  
 Choose S  
 Arrange the S  
 ${}^8C_5 \times 4! \times 2!$   
 Arrange the Z.

$$= \frac{2688}{2688}$$



d)



$$\frac{dv}{dt} = 1.5 \text{ cm/s}$$

$$\frac{x}{7.5} = \frac{r}{5}$$

$$r = \frac{8x}{3}$$

$$V = \frac{\pi r^2 h}{3}$$

$$= \frac{\pi \times \frac{64x^2}{9} \times x}{3}$$

$\Rightarrow$

$$V = \frac{4\pi x^3}{27} \text{ as required.}$$

$$\frac{dv}{dx} = \frac{4\pi}{27} \times 3x^2$$

$$\frac{dv}{dx} = \frac{4\pi x^2}{9}$$

let  $x = 5$

$$\frac{dv}{dx} = \frac{4 \times \pi \times 25}{9}$$

$$= \frac{100\pi}{9}$$

$$\frac{dx}{dt} = \frac{dx}{dv} \times \frac{dv}{dt}$$

$$= \frac{9}{100\pi} \times 1.5$$

$$= 0.042 \dots$$

$$\frac{dx}{dt} = 0.04 \text{ cm/sec}$$

$$13a) \quad \tan 2x + \cot 2x = 2 \operatorname{cosec} 4x$$

$$\text{RHS} = \frac{2}{\sin 4x}$$

$$\text{RHS} = \frac{2 \cdot 1}{2 \sin 2x \cos 2x} \quad (*)$$

$$\text{LHS} = \tan 2x + \frac{1}{\tan 2x}$$

$$= \frac{\sin 2x}{\cos 2x} + \frac{\cos 2x}{\sin 2x}$$

$$= \frac{\sin^2 2x + \cos^2 2x}{\sin 2x \cos 2x}$$

$$= \frac{1}{\sin 2x \cos 2x}$$

$$= \underline{\underline{2 \operatorname{cosec} 4x}} \quad \text{from } (*)$$

$$b) \quad 2 \cos 2\theta + 3 \sin 2\theta$$

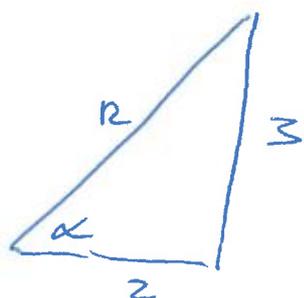
$$R \cos(2\theta - \alpha) = R \cos 2\theta \cos \alpha + R \sin 2\theta \sin \alpha$$

$$R \cos \alpha = 2$$

$$R \sin \alpha = 3$$

$$\cos \alpha = \frac{2}{R}$$

$$\sin \alpha = \frac{3}{R}$$



$$\tan \alpha = \frac{3}{2}$$

$$R = \sqrt{13}$$

$$\underline{\underline{\alpha = 56.19^\circ}}$$

$$2 \cos 2\theta + 3 \sin 2\theta$$

$$= \sqrt{13} \cos(2\theta - 56.19^\circ)$$

$$ii) 2 \cos 2\theta + 3 \sin 2\theta = 1$$

$$\sqrt{13} \cos(2\theta - 56^\circ 19') = 1$$

$$\cos(2\theta - 56^\circ 19') = \frac{1}{\sqrt{13}}$$

$$2\theta - 56^\circ 19' = 73^\circ 54', 286^\circ 6'$$

$$2\theta = 130^\circ 12', 342^\circ 25'$$

↑  
accept 13'

$$\theta = 65^\circ 6', 171^\circ 12'$$


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$$c) 9^{n+2} - 4^n \text{ div by } 5$$

let  $n=1$

$$9^3 - 4$$

$$= 725$$

Is div. by 5. ✓

Assume true for  $n=k$

$$9^{k+2} - 4^k = 5P \quad P \in \mathbb{Z}$$

Let  $n=k+1$

$$9^{k+3} - 4^{k+1}$$

$$9^{k+3} - 4 \times 4^k$$

$$9^{k+3} - 4(9^{k+2} - 5P)$$

$$9^{k+3} - 4 \times 9^{k+2} + 20P$$

$$9^{k+2}(9-4) + 20P$$

$$5 \times 9^{k+2} + 20P$$

$$5(9^{k+2} + 4P)$$

∴ Div by 5.

∴ ~~True~~ If it is true for  $n=k$ , it is also true for  $n=k+1$ . As it is true for  $n=1$ , it is true for  $n=2, 3, \dots$

$$d) \frac{d}{dx} (\tan^{-1} x)$$

$$= \frac{1}{1+x^2}$$

$$\frac{d}{dx} \left[ -\tan^{-1} \frac{1}{x} \right]$$

$$= \frac{-x \left[ \frac{1}{x} - x \frac{1}{x^2} \right]}{1 + \left( \frac{1}{x} \right)^2}$$

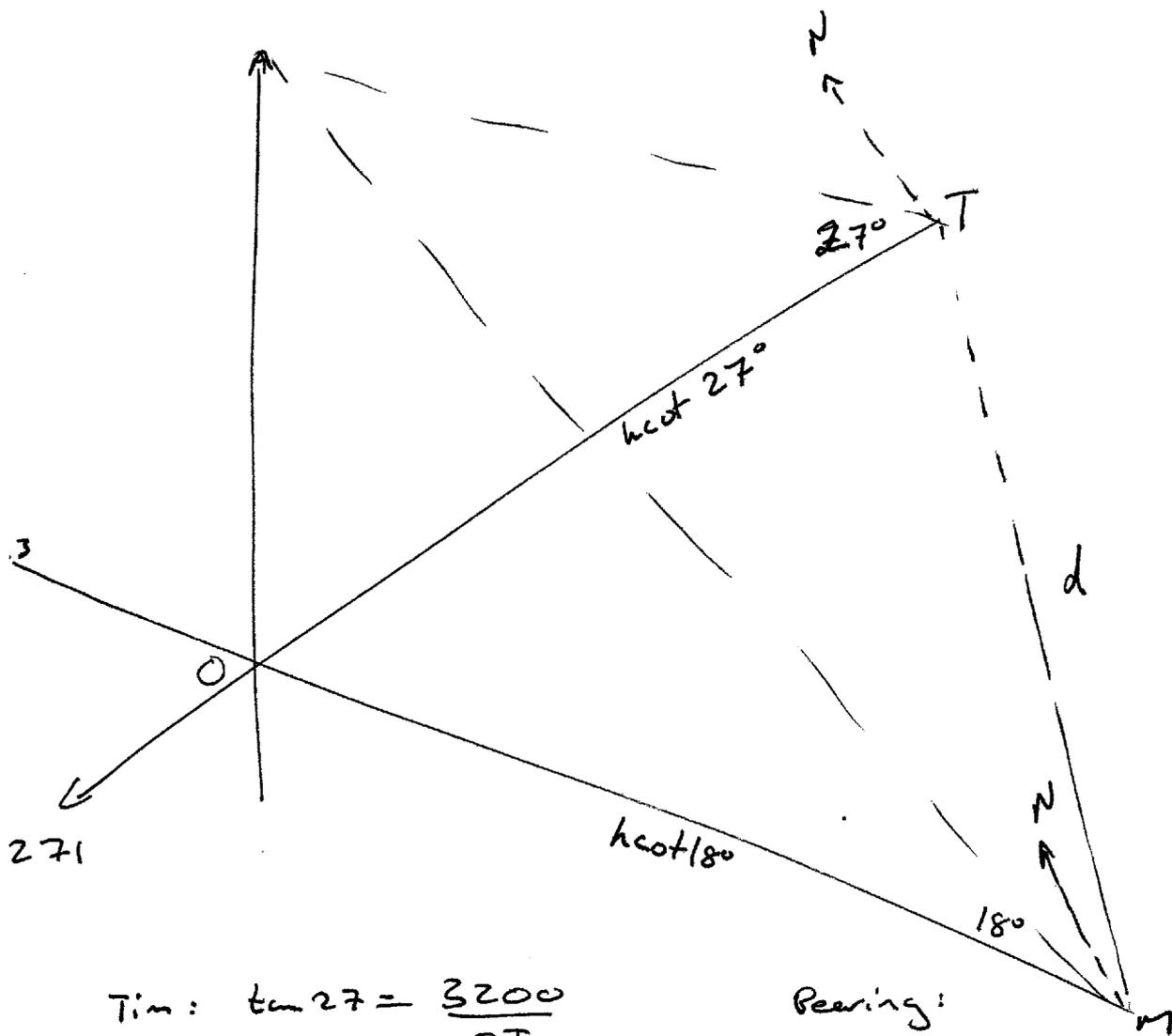
$$= \frac{-x \left[ \frac{1}{x} - \frac{1}{x} \right]}{1 + \left( \frac{1}{x} \right)^2}$$

$$= \frac{0}{1 + \left( \frac{1}{x} \right)^2}$$

$$= \frac{1}{x^2 + 1}$$

$$= \frac{1}{1+x^2}$$

$$= \underline{\underline{\frac{d}{dx} \tan^{-1} x}}$$



Tim:  $\tan 27 = \frac{3200}{OT}$

$OT = 3200 \cot 27^\circ$

Michael:  $OM = 3200 \cot 18^\circ$

Bearing:

$323 - 271 = 52^\circ$

$$d^2 = 3200^2 \cot^2 27 + 3200^2 h^2 \cot^2 18 - 2 \times 3200 \cot 27 \times 3200 \cot 18 \cos 52^\circ$$

i)  $d = \sqrt{3200^2 (\cot^2 27 + \cot^2 18 - \cot 27 \cot 18 \cos 52^\circ)}$

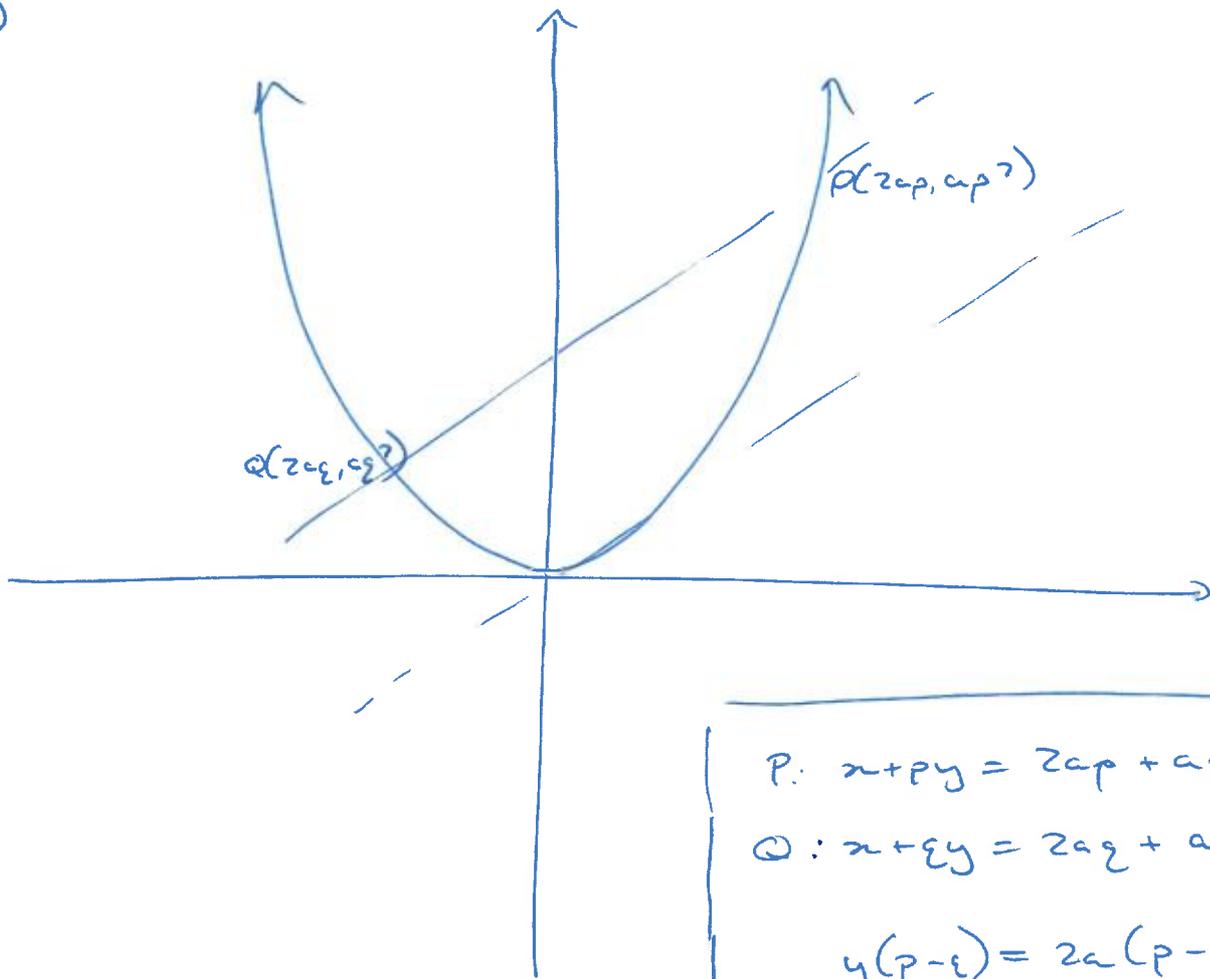
ii)  $= \sqrt{3200^2 (\tan^2 63^\circ + \tan^2 72^\circ - \tan 63^\circ \tan 72^\circ \cos 52^\circ)}$

$d = 7764 \text{ m}$

iii)  $\frac{\sin 52}{7764} = \frac{\sin \theta}{3200 \tan 63}$

$\theta = 39^\circ 36'$   
 $\therefore$  Bearing from M to T is  $323 + 39^\circ 36' = 003^\circ \text{T}$ .

14a)



$$m_{PQ} = \frac{ap^2 - aq^2}{2ap - 2aq}$$

$$1 = \frac{a(p+q)(p-q)}{2a(p-q)}$$

$$1 = \frac{p+q}{2}$$

$$2 = p+q$$

$$P: x+py = 2ap + ap^3$$

$$Q: x+qy = 2aq + aq^3$$

$$y(p-q) = 2a(p-q) + a(p^3-q^3)$$

$$y = a[z + p^2 + pq + q^2]$$

~~$$y = p+q$$~~

$$x + ap(z + p^2 + pq + q^2) = 2ap + ap^3$$

$$x + ap^2q + apq^2 = 0$$

~~$$x = -a(p^2+q^2)$$~~

$$x = -apq(p+q)$$

Intersection:  $(-apq(p+q), 2+p^2+pq+q^2)$

$$x = -apq(p+q)$$

$$x = -2apq$$

$$y = 2 + p^2 + pq + q^2$$

$$y = pq + 2 + q^2 + p^2$$

$$= \frac{x}{2a} + 2 + q^2 + p^2$$

straight  
line with  
gradient  
-1  
2a

$$y = -\frac{x}{2a} + 2 + p^2 + q^2$$

$$= -\frac{x}{2a} + (p+q)^2 - 2pq + 2$$

$$= -\frac{x}{2a} + 4 + 2 + \frac{x}{a}$$

$$y = \frac{x}{2a} + 6$$

$\therefore$  straight line, gradient  ~~$\frac{1}{2a}$~~   $\frac{1}{2a}$

b)



Show:  $x = 56t \cos \theta$

$y = -4.9t^2 + 56t \sin \theta$

Horizontal

$\ddot{x} = 0$

$\dot{x} = 56 \cos \theta \quad (c = 56 \cos \theta)$

$x = 56t \cos \theta + c$

$t = 0 \quad x = 0$

$\therefore c = 0$

$x = 56t \cos \theta$

$\ddot{y} = -9.8$

$\dot{y} = -9.8t + c$

$t = 0 \quad \dot{y} = 56 \sin \theta$

$\dot{y} = -9.8t + 56 \sin \theta$

$y = -4.9t^2 + 56t \sin \theta + c$

$t = 0 \quad y = 0 \quad c = 0$

$y = -4.9t^2 + 56t \sin \theta$

ii) let  $x = 70, y = 15$

$70 = 56t \cos \theta$

$t = \frac{70}{56 \cos \theta}$

~~15~~  $15 = -4.9t^2 + 56t \sin \theta$

$15 = -4.9 \left( \frac{70}{56 \cos \theta} \right)^2 + 56 \times \frac{70}{56 \cos \theta} (\sin \theta)$

$= \frac{-4.9 \times 70}{56} \times (1 + \tan^2 \theta) + 70 \tan \theta$

$$15 = \cancel{6.125} - \frac{49}{8} - \frac{49}{8} \tan^2 \theta + 70 \tan \theta$$

$$120 = -49 - 49 \tan^2 \theta + 560 \tan \theta$$

$$0 = 49 \tan^2 \theta - 560 \tan \theta + 169$$

$$\tan \theta = \frac{560 \pm \sqrt{560^2 - 4 \times 49 \times 169}}{2 \times 49}$$

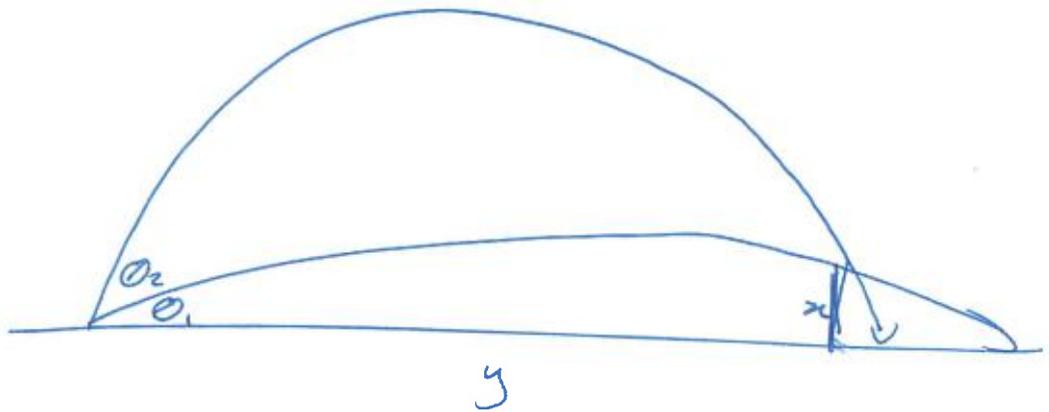
$$\tan \theta = 11.12, 0.31$$

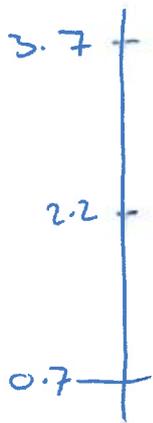
$$\theta = 84^\circ 52', 17^\circ 14'$$

iii)  $45^\circ$  will give max range

Anything other than that angle, ~~for~~ there will always be 2 answers.

ie





Period 6 hours 10 mins.  
 370 mins for half period  
 $\therefore$  740 for whole

~~$\omega =$~~   
 Period =  $\frac{2\pi}{\omega}$  ← Must state

$\omega = \frac{2\pi}{370 \times 2} = \frac{2\pi}{740}$  ← show.

$= \frac{\pi}{370}$

$x = -a \cos \omega t + 2.2$

$x = -1.5 \cos \frac{\pi t}{370} + 2.2$

$z = -1.5 \cos \frac{\pi t}{370} + 2.2$

$\frac{-0.2}{-1.5} = \frac{\cos \frac{\pi t}{370}}{1}$

$t = 169 \sqrt{2} \text{ mins } 15'$

$t = 8:55 + 169 \text{ mins}$   
 $= 11:55 - 11 \text{ mins}$

11:44 am